Nirma University

Institute of Technology

Semester End Examination (RPR), April - 2025
B. Tech. in All Programmes, Semester-I
1MH101CC22 / 1MH101 Mathematics I

Roll/Exam No.:________________ Supervisor's initial ________________ with date

Time: 3 Hours **Max. Marks: 100**

Instructions:
1. Attempt all questions.
2. Figures to the right indicate full marks.
3. Use Section-wise separate answer book.

Section-I

Q.1 **Answer the following:** **(18)**

(A) CLO1, BL3 Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & -3 & 1 & -6 \\ 1 & 1 & 2 & -1 & 7 \\ 2 & 2 & -3 & 1 & -3 \\ 0 & 2 & 1 & 2 & -2 \end{bmatrix}$ by reducing it into row echelon form. (06)

(B) CLO1, BL3 Find the value of α and β, for which the system of linear equations $x + 2y + z = 8$, $2x + 2y + 2z = 13$, $3x + 4y + \alpha z = \beta$ has infinitely many solutions. (06)

(C) CLO1, BL3 Solve: $3x + y + z + w = 0$, $5x - y + z + w = 0$. (06)

Q.2 **Answer the following:** **(18)**

(A) CL02, BL3 Which of the following sets of polynomials are linearly independent or dependent? (06)
(i) $S_1 = \{1 + 2x + 5x^2, 3 + 6x + 5x^2\}$ in P_2
(ii) $S_2 = \{-3 + 4x^2, 5, 5 - x + 2x^2, 1 + x + 5x^2\}$ in P_2.

(B) CLO2, BL3 Check whether the set S = {(1, 0, 0), (0, 1, 0), (2, 0, 0), (0, 0, -3)} is span set of the Vector Space R^3. (06)

(C) CLO2, BL3	Extend a set $S=\{(-1,2,3),(1,-2,-2)\}$ to obtain a basis for the Vector Space R_3.	(06)
Q.3	**Answer the following:**	**(14)**
(A) CLO1, BL3	Find the Inverse of the given matrix $$\begin{bmatrix} 3 & 1 & -3 & -8 \\ 4 & 2 & -1 & -3 \\ 1 & 2 & 4 & 9 \\ 2 & 4 & 3 & 3 \end{bmatrix}$$ by using Gauss Gauss-Jordan Method if possible if possible.	(07)
(B) CLO2, BL3	For the given bases $B=\{(1,0,0),(0,1,0),(0,0,1)\}$ and $C=\{(1,-1,1),(0,1,2),(3,0,-1)\}$ of R^3 (i) Compute $[u]_B$ if $[u]_C=(9,-1,-8)$ & (ii) Compute $[u]_C$ if $[u]_B=(-6,7,2)$.	(07)

Section-II

Q.4	**Answer the following**	**(18)**
(A) CLO3, BL3	Consider the basis $S=\{V_1,V_2\}$ for R^2 and $T:R^2\to P_2$ be the linear transformation such that $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)=2-3x+x^2$ and $T\left(\begin{bmatrix}2\\3\end{bmatrix}\right)=1-x^2$. Find $T\begin{bmatrix}a\\b\end{bmatrix}$ and hence compute $T\begin{bmatrix}-1\\2\end{bmatrix}$.	(06)
(B) CLO3, BL3	Find the standard matrix of the linear operator $T:R^2\to R^2$ that first dilates a vector with factor $k=2$, then rotates the resulting vector by an angle of 45° about y-axis.	(06)
(C) CLO3, BL3	Find the basis for kernel and range of the linear transformation $T:M_{22}\to M_{22}$ defined by $T\left(\begin{bmatrix}a & b\\ c & d\end{bmatrix}\right)=\begin{bmatrix}a+b & b+c\\ a+d & b+d\end{bmatrix}$.	(06)
Q.5	**Solve the following**	**(18)**
(A) CL04, BL3	Determine algebraic and geometric multiplicity of each eigenvalue and corresponding eigen vector for the matrix $\begin{bmatrix}1 & 2 & 2\\ 0 & 2 & 1\\ -1 & 2 & 2\end{bmatrix}$.	(06)
(B) CLO4, BL3	Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix}2 & 4\\ 1 & 2\end{bmatrix}$ and using it find A^{-1} and A^3-5A^2.	(06)

(C) CLO4, BL3

Find a matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and hence find A^{13}. (06)

Q.6 **Solve the following** **(14)**

(A) CLO3, BL3

Consider the basis $S_1 = \{u_1, u_2\}$ and $S_2 = \{v_1, v_2\}$ for R^2 , where $u_1 = (1,0)$, $u_2 = (0,1)$, $v_1 = (2,1)$ and $v_2 = (-3,4)$. (07)

(i) Find the transition matrix from S_2 to S_1.

(ii) Find the transition matrix from S_1 to S_2.

(B) CLO4, BL3

Show that the matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ is similar to diagonal matrix and hence find its diagonal and modal matrix. (07)

Nirma University

Institute of Technology

Semester End Examination (RPR), June - 2023

B. Tech. in CL / ME / EE / EI / EC / CSE, Semester-I

Int. B. Tech. (CSE) - MBA, Semester-I

1MH101 Mathematics I

Roll / Exam No.		Supervisor's initial with date	

Time: 3 Hours — Max. Marks: 100

Instructions:
1. All questions are compulsory.
2. Figures to right indicate full marks.
3. Use Section-wise separate answer book.
4. Scientific calculator is allowed.

SECTION I

Q-1. Solve the following

[A] CO1, L2 — Find the rank of the following matrix by reducing to Row Echelon form. **[6]**

$$A=\begin{bmatrix}1 & 2 & -1 & 3\\ 3 & 4 & 0 & -1\\ -1 & 0 & -2 & 7\end{bmatrix}$$

[B] CO1, L3 — Determine the values of λ for which the following system is consistent. Also, solve the system for these values of λ. **[6]**

$x+2y+z=3,\ \ x+y+z=\lambda,\ \ 3x+y+3z=\lambda^2.$

[C] CO1, L2 — Find the inverse of matrix $A=\begin{bmatrix}5 & -1 & 5\\ 0 & 2 & 0\\ -5 & 3 & -15\end{bmatrix}$ using Gauss Jordan method. **[6]**

Q-2. Solve the following

[A] CO2, L3 — Check whether the set $W=\{(x,y)\mid xy\geq 0\}$ is a subspace of R^2. **[6]**

[B] CO2, L3 — Determine whether the set of all 2×2 matrices of the form $\begin{bmatrix}a & 1\\ 1 & b\end{bmatrix}$ with the standard addition and standard multiplication is a vector space. **[6]**

[C] CO2, L2 — Check whether the given set of vectors is linearly dependent or independent. $S=\{(-2,0,1),(3,2,5),(6,-1,1),(7,0,-2)\}$ **[4]**

Q-3. Solve the following

[A] CO1, L2 — Examine for a trivial solution for the following system. **[6]**

$2x_1+x_2+3x_3=0,\ \ x_1+2x_2=0,\ \ x_2+x_3=0$

[B] CO2, L2 — For which value of λ will the vector $v=(1,\lambda,5)$ be the linear combination of vectors $v_1=(1,-3,2)$ and $v_2=(2,-1,1)$? **[6]**

[C] CO2, L3 — Find a basis for the subspace of P_2 spanned by the vectors $1+x, x^2, -2+2x^2, -3x$. **[4]**

SECTION II

Q-4. **Solve the following**

[A] CO3, L3

Consider a basis $S=\left\{\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix},\begin{bmatrix}0 & -1\\1 & 1\end{bmatrix},\begin{bmatrix}1 & -1\\0 & 0\end{bmatrix},\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\right\}$ for $M_{2,2}$ and T: $M_{2,2} \rightarrow R^3$ be the linear transformation such that $T\left(\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right)=\begin{pmatrix}1\\2\\1\end{pmatrix}$, $T\left(\begin{bmatrix}0 & -1\\1 & 1\end{bmatrix}\right)=\begin{pmatrix}1\\-2\\1\end{pmatrix}$, $T\left(\begin{bmatrix}1 & -1\\0 & 0\end{bmatrix}\right)=\begin{pmatrix}1\\-2\\-1\end{pmatrix}$ and $T\left(\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\right)=\begin{pmatrix}-1\\2\\1\end{pmatrix}$. Find the linear transformation and use that to find $T\left(\begin{bmatrix}1 & 0\\4 & 2\end{bmatrix}\right)$. **[6]**

[B] CO3, L3

Let $T: R^3 \rightarrow R^2$ be the linear transformation defined by **[6]**

$$T(x,y,z) = (x + 2y - z, y + z, x + 2y - z)$$

Find the Kernel and nullity of T.

[C] CO3, L2

Find the standard matrix for the linear operators on R^2 with reflection about x axis followed by rotation of 45^0. **[6]**

Q-5. **Solve the following**

[A] CO3, L3

Let $T: R^2 \rightarrow R^3$ be the linear transformation. Let $A=\begin{bmatrix}1 & 2\\2 & 3\\3 & 4\end{bmatrix}$ be the matrix representation of the linear transformation with respect to the ordered basis $B=\left(\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}3\\4\end{bmatrix}\right)$ in R^2 and $C=\left(\begin{bmatrix}-1\\1\\1\end{bmatrix},\begin{bmatrix}1\\-1\\1\end{bmatrix},\begin{bmatrix}1\\1\\-1\end{bmatrix}\right)$ in R^3. Then, determine the linear transformation T and hence find $T\left(\begin{bmatrix}-3\\2\end{bmatrix}\right)$. **[7]**

[B] CO3, L3

Let $T: R^3 \rightarrow R^3$ be a linear transformation defined by **[7]**

$$T(a,b,c) = (a - 2b, 3a + b + c, 4a)$$

Is T invertible? If so, find the inverse of the linear transformation.

Q-6. **Solve the following**

[A] CO4, L2

Find the model matrix P that diagonalize the matrix $A=\begin{bmatrix}8 & -6 & 2\\-6 & 7 & -4\\2 & -4 & 3\end{bmatrix}$. **[6]**

[B] CO4, L3

For $A=\begin{bmatrix}2 & 1 & -1\\1 & 0 & -1\\1 & 1 & 2\end{bmatrix}$ find A^{-1} using Caley-Hamilton theorem. **[6]**

[C] CO4, L3

Find the algebraic multiplicity and geometric multiplicity of the eigen values of the matrix A, where $A=\begin{bmatrix}3 & 2 & 4\\2 & 0 & 2\\4 & 2 & 3\end{bmatrix}$. **[6]**

Nirma University

Institute of Technology

Semester End Examination (RPR), May - 2024

B. Tech. in CL / ME / EE / EC / EI / CSE, Semester-I

Int. B. Tech. (CSE) - MBA, Semester-I

1MH101 Mathematics I

Exam No.________________ Supervisor's initial with date

Time: 3 Hours. Max. Marks: 100

Instructions:

1. Attempt all questions.
2. Figures to the right indicate full marks.
3. Use Section wise Separate answer book.

Section A

Q.1 Answer the following: **(18)**

(A) CO1, L3 Find the rank of matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing into row echelon form. (06)

(B) CO1, L3 Find the value of a and b for which the system of linear equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + az = b$ has (a) unique solution, (b) infinitely many solutions & (c) no solution. (06)

(C) CO1, L3 Solve: x + 2y - z = 3, 3x - y - 2z = 1, 2x - 2y + 3z = 2, x - y + z = -1. (06)

Q.2 Answer the following: **(18)**

(A) CO2, L3 Give the answers with proper justification: (06)

(i) Is V, the set consisting of all ordered pairs of real numbers (x, y), where $x \leq 0$, with the usual operations a vector space?

(ii) Is the set W consisting of all planes not passing through the origin Sub Space of vector Space R^3?

(B) CO2, L3 Determine whether the set of vectors $3x + 2x^2 + x^3$, $1 + 2x^3$, $4 + 6x + 8x^2 + 6x^3, 1 + x + 2x^2 + x^3$ form a basis for P_3 or not. (06)

(C) CO2, L3 Extend a set $\{(1, -4,2, -3), (-3,8, -4,6)\}$ to obtain a basis for the Vector Space R^4. (06)

Q.3 Answer the following: **(14)**

(A) CO1, L3 Find the inverse for matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 1 & -1 & -2 \\ -4 & -2 & -3 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ if possible. (07)

(B) CO2, L3 For the bases S = {(1,2),(0,1)} and T = {(1,1),(2,3)} of R^2 (i) Find the transition matrix from S to T, (ii) Find the transition matrix from T to S, (iii) Compute $[w]_S$ for w = (1,5). (07)

Section B

Q.4 Answer the following: **(18)**

(A) CO3, L3 Consider the basis S = {(-2,1), (1,3)} for R^2 and $T : R^2 \rightarrow R^3$ be the Linear Transformation such that T(-2,1) = (−1,2,0) and T(1,3) = (0,-3,5). Find the formula for T(a, b) and use it to find T(2,-3). (06)

(B) CO3, L3 Find the standard matrix of the composition of a rotation of 45^0 about y-axis, followed by a dilation with the factor $k = \sqrt{2}$ in R^3. (06)

(C) CO3, L3 Find the kernel and range for the Linear Transformation $T: R^3 \rightarrow R^3$, defined by $T(x, y, z) = (x + 2y - z,\ y + z, x + y - 2z)$ and hence verify dimension theorem for it. (06)

Q.5 Answer the following: **(18)**

(A) C04, L3 For the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, find the Algebraic Multiplicity & Geometric Multiplicity of each Eigen values and corresponding Eigen vectors. (06)

(B) CO4, L3 For $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ find A^4 and A^{-1} using Caley-Hamilton theorem. (06)

(C) CO4, L3 Find a matrix P that Diagonalize $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, and hence find A^5. (06)

Q.6 Answer the following: **(14)**

(A) CO3, L3 Let $T: R^2 \rightarrow R^2$, defined by $T(x, y) = (x - 2y, -x)$ and $B = \{(1, 0), (0, 1)\}$ & $B' = \{(2, 1), (-3, 4)\}$ are basis of R^2. Then using $[T(v)]B' = P^{-1} [T(v)]B\, P$, find $[T(v)]B'$, where P is the transition matrix from B' to B. (07)

(B) CO4, L3 Diagonalize the matrix $A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ if possible. (07)

Nirma University

Institute of Technology

Supplementary Examination (SPE), February - 2024

B. Tech. in All Programmes, Semester-I

1MH101 Mathematics I

Exam No.________________ Supervisor's initial with date

Time: 3 Hours. Max. Marks: 100

Instructions:

1. Attempt all questions.
2. Figures to the right indicate full marks.
3. Use Section wise Separate answer book.

Section A

Q.1 Answer the following: **(18)**

(A) CO1, L3 Find the rank of matrix $\begin{bmatrix} 1 & 2 & -3 & 1 & -6 \\ 1 & 1 & 2 & -1 & 7 \\ 2 & 2 & -3 & 1 & -3 \\ 0 & 2 & 1 & 2 & -2 \end{bmatrix}$ by reducing into row echelon form. (06)

(B) CO1, L3 Find the value of a and b for which the system of linear equations (06)

$$x + 2y + z = 8,\ 2x + 2y + 2z = 13,\ 3x + 4y + az = b$$

has (a) unique solution, (b) infinitely many solutions & (c) no solution.

(C) CO1, L3 Solve : $y + 3z - 2w = 0$, $2x + y - 4z + 3w = 0$, $2x + 3y + 2z - w = 0$, $-4x - 3y + 5z - 4w = 0$. (06)

Q.2 Answer the following: **(18)**

(A) CO2, L3 Give the answers with proper justification: (06)

(i) Is V, the set consisting of all n x n singular matrices together with standard matrix addition and scalar multiplication a vector space?

(ii) Is the set W consisting of all planes passing through the origin Sub Space of vector Space R^3?

(B) CO2, L3 Check whether $M = \left\{ \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ is a basis for the vector space $M_{22} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a,b,c,d \in R \right\}$? (06)

(C) CO2, L3 Extend the set $S = \{x^3 - 4x^2 + 2x - 3, -3x^3 + 8x^2 - 4x + 6\}$ to the basis of vector space P_3. (06)

Q.3 Answer the following: **(14)**

(A) CO1, L3 Find the inverse for matrix $A = \begin{bmatrix} 3 & 1 & -3 & -8 \\ 4 & 2 & -1 & -3 \\ 1 & 2 & 4 & 9 \\ 2 & 4 & 3 & 3 \end{bmatrix}$ if possible. (07)

(B) CO2, L3 For the bases S = {(-3,2),(4,-2)} and T = {(-1,2),(2,-2)} of R^2 (i) Find the transition matrix from S to T, (ii) Find the transition matrix from T to S, (iii) Compute $[w]_S$ for w = (1,2) & (iv) Compute $[w]_T$ using $[w]_S$. (07)

Section B

Q.4 Answer the following: **(18)**

(A) CO3, L3 Determine the linear transformation T: $M_{22} \rightarrow R$, whose images of basis vectors are given by $T\left\{\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\right\}=3$, $T\left\{\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}\right\}=-4$, $T\left\{\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\right\}=1$, $T\left\{\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\right\}=-1$. Hence compute $T\left\{\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}\right\}$. (06)

(B) CO3, L3 Find the standard matrix of the composition of a rotation of 30^0 about x-axis, followed by a rotation of 30^0 about the z-axis, followed by a contraction with the factor k = 1/4 in R^3. (06)

(C) CO3, L3 Find the kernel and range for the Linear Transformation T: $R^2 \rightarrow R^3$, defined by T (x, y) = (13x - 9y, -x - 2y, -11x - 6y) and hence verify dimension theorem for it. (06)

Q.5 Answer the following: **(18)**

(A) C04, L3 For the matrix $A=\begin{bmatrix}-3 & -7 & -5\\2 & 4 & 3\\1 & 2 & 2\end{bmatrix}$ find the Algebraic Multiplicity & Geometric Multiplicity of each Eigen values and corresponding Eigen vectors. (06)

(B) CO4, L3 Verify Caley-Hamilton theorem for $A=\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}$ then using it find the matrix of $A^5 - 3A^4 + A^3 - 7A^2 + 5A + I$. (06)

(C) CO4, L3 Find a matrix P that Diagonalize $A=\begin{bmatrix}4 & 1\\3 & 2\end{bmatrix}$, and hence find A^5. (06)

Q.6 Answer the following: **(14)**

(A) CO3, L3 Let T: $R^2 \rightarrow R^2$, defined by T (x,y) = (x - y, -y) and let B = {(1,0),(0,1)} & C = {(2,1),(-3,4)}. Then using $[T]_C = P^{-1}[T]_B P$, find $[T]_C$, where P is the transition matrix from C to B. (07)

(B) CO4, L3 Diagonalize the matrix $A=\begin{bmatrix}2 & -1 & 1\\-1 & 2 & -1\\1 & -1 & 2\end{bmatrix}$ if possible. (07)

Nirma University

Institute of Technology

Supplementary Examination (SPE), March - 2024

B. Tech. (All Programmes), Semester-I

1MH101 Mathematics I

Exam No.________________ Supervisor's initial with date

Time: 3 Hours. Max. Marks: 100

Instructions:

1. Attempt all questions.
2. Figures to the right indicate full marks.
3. Use Section wise Separate answer book.

Section A

Q.1 Answer the following: **(18)**

(A) CO1, L3 Find the rank of matrix $\begin{bmatrix} 3 & 2 & -3 & 1 & -6 \\ -1 & 3 & 2 & -2 & 5 \\ 2 & 2 & -3 & 1 & -1 \\ 0 & 1 & 4 & 2 & -2 \end{bmatrix}$ by reducing into row echelon form. (06)

(B) CO1, L3 Find the value of a and b for which the system of linear equations $x + 2y + z = 8,\ 2x + 2y + 2z = 13,\ 3x + 4y + az = b$ has (a) unique solution, (b) infinitely many solutions & (c) no solution. (06)

(C) CO1, L3 Solve : $x - 2y + 3z - 2w = 0$, $2x + y - 4z + 3w = 0$, $x + 3y + 2z - w = 0$, $-4x - y + 2z - 4w = 0$. (06)

Q.2 Answer the following: **(18)**

(A) CO2, L3 Give the answers with proper justification: (06)

(i) Is V, the set consisting of all n x n symmetric matrices together with standard matrix addition and scalar multiplication a vector space?

(ii) Is the set W consisting of all planes not passing through the origin Sub Space of vector Space R^3?

(B) CO2, L3 Check whether $M = \left\{\begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}\right\}$ is a basis for the vector space $M_{22} = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a,b,c,d \in R\right\}$? (06)

(C) CO2, L3 Extend the set $S = \{x^3 - 2x^2 + 4x - 3,\ 3x^3 + 5x^2 - 2x + 4\}$ to the basis of vector space P_3. (06)

Q.3 Answer the following: **(14)**

(A) CO1, L3 Find the inverse for matrix $A = \begin{bmatrix} 1 & -1 & 3 & -2 \\ 4 & 3 & -1 & -3 \\ 3 & 2 & -4 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix}$ if possible. (07)

(B) CO2, L3 For the bases S = {(-3,2), (4,-2)} and T = {(-1,2), (2,-2)} of R^2 (i) Find the transition matrix from S to T, (ii) Find the transition matrix from T to S, (iii) Compute $[w]_S$ for w = (1,2) & (iv) Compute $[w]_T$ using $[w]_S$. (07)

Section B

Q.4 Answer the following: **(18)**

(A) CO3, L3 Determine the linear transformation T: $M_{22} \rightarrow R$, whose images of basis vectors are given by $T\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right\} = 2, T\left\{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\right\} = -1, T\left\{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right\} = 3, T\left\{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right\} = 4$. Hence compute $T\left\{\begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}\right\}$. (06)

(B) CO3, L3 Find the standard matrix of the composition of a rotation of 30^0 about y-axis, followed by a dilation with the factor k = $\sqrt{3}$ in R^3. (06)

(C) CO3, L3 Find the kernel and range for the Linear Transformation T: $R^2 \rightarrow R^3$, defined by T (x, y) = (3x - 4y, x - 3y, 2x - 6y) and hence verify dimension theorem for it. (06)

Q.5 Answer the following: **(18)**

(A) C04, L3 For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ find the Algebraic Multiplicity & Geometric Multiplicity of each Eigen values and corresponding Eigen vectors. (06)

(B) CO4, L3 Verify Caley-Hamilton theorem for $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, then using it find the matrix of $A^5 - 3A^4 + A^3 - 7A^2 + 5A + I$. (06)

(C) CO4, L3 Find a matrix P that Diagonalize $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and hence find A^5. (06)

Q.6 Answer the following: **(14)**

(A) CO3, L3 Let T: $R^2 \rightarrow R^2$, defined by T (x, y) = (2x - y, y) and B = {(1,0),(0,1)}, C = {(1,2),(-4,3)} are the basis of R^2. Then using $[T]_C = P^{-1} [T]_B P$, find $[T]_C$, where P is the transition matrix from C to B. (07)

(B) CO4, L3 Diagonalize the matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ if possible. (07)

Nirma University

Institute of Technology

Semester End Examination (RPR), April - 2025

B. Tech. in All Programmes, Semester-I

1MH101CC22 / 1MH101 Mathematics I

Roll/Exam No.:________________ Supervisor's initial with date ________________

Time: 3 Hours **Max. Marks: 100**

Instructions:

1. Attempt all questions.
2. Figures to the right indicate full marks.
3. Use Section-wise separate answer book.

Section-I

Q.1 **Answer the following:** **(18)**

(A) CLO1, BL3 Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & -3 & 1 & -6 \\ 1 & 1 & 2 & -1 & 7 \\ 2 & 2 & -3 & 1 & -3 \\ 0 & 2 & 1 & 2 & -2 \end{bmatrix}$ by reducing it into row echelon form. (06)

(B) CLO1, BL3 Find the value of α and β, for which the system of linear equations $x + 2y + z = 8$, $2x + 2y + 2z = 13$, $3x + 4y + \alpha z = \beta$ has infinitely many solutions. (06)

(C) CLO1, BL3 Solve: $3x + y + z + w = 0$, $5x - y + z + w = 0$. (06)

Q.2 **Answer the following:** **(18)**

(A) CL02, BL3 Which of the following sets of polynomials are linearly independent or dependent? (06)

(i) $S_1 = \{1 + 2x + 5x^2, 3 + 6x + 5x^2\}$ in P_2

(ii) $S_2 = \{-3 + 4x^2, 5, 5 - x + 2x^2, 1 + x + 5x^2\}$ in P_2.

(B) CLO2, BL3 Check whether the set S = {(1, 0, 0), (0, 1, 0), (2, 0, 0), (0, 0, -3)} is span set of the Vector Space R^3. (06)

(C) CLO2, BL3 — Extend a set $S = \{(-1,2,3),(1,-2,-2)\}$ to obtain a basis for the Vector Space R_3. (06)

Q.3 **Answer the following:** **(14)**

(A) CLO1, BL3 — Find the Inverse of the given matrix

$$\begin{bmatrix} 3 & 1 & -3 & -8 \\ 4 & 2 & -1 & -3 \\ 1 & 2 & 4 & 9 \\ 2 & 4 & 3 & 3 \end{bmatrix}$$

(07)

by using Gauss Gauss-Jordan Method if possible if possible.

(B) CLO2, BL3 — For the given bases $B = \{(1,0,0),(0,1,0),(0,0,1)\}$ and $C = \{(1,-1,1),(0,1,2),(3,0,-1)\}$ of R^3

(i) Compute $[u]_B$ if $[u]_C = (9,-1,-8)$ & (07)

(ii) Compute $[u]_C$ if $[u]_B = (-6,7,2)$.

Section-II

Q.4 **Answer the following** **(18)**

(A) CLO3, BL3 — Consider the basis $S = \{V_1, V_2\}$ for R^2 and $T: R^2 \to P_2$ be the linear transformation such that $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = 2 - 3x + x^2$ and $T\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = 1 - x^2$. Find $T\begin{bmatrix}a\\b\end{bmatrix}$ and hence compute $T\begin{bmatrix}-1\\2\end{bmatrix}$. (06)

(B) CLO3, BL3 — Find the standard matrix of the linear operator $T: R^2 \to R^2$ that first dilates a vector with factor $k = 2$, then rotates the resulting vector by an angle of 45° about y-axis. (06)

(C) CLO3, BL3 — Find the basis for kernel and range of the linear transformation $T: M_{22} \to M_{22}$ defined by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b & b+c \\ a+d & b+d \end{bmatrix}$. (06)

Q.5 **Solve the following** **(18)**

(A) CL04, BL3 — Determine algebraic and geometric multiplicity of each eigenvalue and corresponding eigen vector for the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$. (06)

(B) CLO4, BL3 — Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ and using it find A^{-1} and $A^3 - 5A^2$. (06)

(C) CLO4, BL3 — Find a matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and hence find A^{13}. (06)

Q.6 Solve the following **(14)**

(A) CLO3, BL3 — Consider the basis $S_1 = \{u_1, u_2\}$ and $S_2 = \{v_1, v_2\}$ for R^2, where $u_1 = (1,0)$, $u_2 = (0,1)$, $v_1 = (2,1)$ and $v_2 = (-3,4)$. (07)

(i) Find the transition matrix from S_2 to S_1.

(ii) Find the transition matrix from S_1 to S_2.

(B) CLO4, BL3 — Show that the matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ is similar to diagonal matrix and hence find its diagonal and modal matrix. (07)

Nirma University

Institute of Technology

Supplementary Examination (SPE), July - 2025
B. Tech. in CL/CH/ME/CSE, Semester-I
1MH101CC22 Mathematics - I

Roll/Exam No.:________________ Supervisor's initial with date ________________

Time: 3 Hours Max. Marks: 100

Instructions:

1. Attempt all questions.
2. Figures to the right indicate full marks.
3. Use Section-wise separate answer book.

Section-I

Q.1 **Answer the following:** **(18)**

(A) CLO1, BL3 — Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 7 & 9 & 11 & 13 & 15 \\ 14 & 18 & 22 & 26 & 30 \end{bmatrix}$ by reducing into row echelon form. (06)

(B) CLO1, BL3 — Find the value of α and β, for which the system of linear equations $x + 2y + z = 8, 2x + 2y + 2z = 13, 3x + 4y + \alpha z = \beta$ has a unique solution. (06)

(C) CLO1, BL3 — Solve: $y + 3z - 2w = 0,\ 2x + y - 4z + 3w = 0,\ 2x + 3y + 2z - w = 0,\ -4x - 3y + 5z - 4w = 0.$ (06)

Q.2 **Answer the following:** **(18)**

(A) CL02, BL3 — Express $v = (1, -2, 5)$ as a linear combination of vectors $v_1 = (1,1,1), v_2 = (1,2,3)$ *and* $v_3 = (2, -1,1)$ if possible. (06)

(B) CLO2, BL3 — Determine whether $S = \{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ is a basis for the vector space R^3. (06)

(C) CLO2, BL3 — Reduce a set $S = \{1 + x, x^2, -2 + 2x^2, -3x\}$ to obtain a basis for the subspace of P_2. (06)

Q.3 **Answer the following:** **(14)**

(A) CLO1, BL3

Find the Inverse of matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix}$ (07)

by using Gauss Gauss-Jordan Method if possible if possible.

(B) CLO2, BL3

For the bases $S = \{(2,-2),(6,3)\}$ and $T = \{(1,1),(2,3)\}$ of R^2
(i) Find the transition matrix from S to T,
(ii) Find the transition matrix from T to S,
(iii) Compute $[w]_S$ for $w = (3,-5)$ &
(iv) Compute $[w]_T$ using $[w]_S$. (07)

Section-II

Q.4 **Answer the following** **(18)**

(A) CLO3, BL3

Determine the linear transformation $T: M_{22} \to R$ whose image of basis vectors are given by $T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = 1$, $T\left(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\right) = 2$, $T\left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\right) = 3$ and $T\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right) = 4$. Hence, compute $T\left(\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}\right)$. (06)

(B) CLO3, BL3

Find the standard matrix of the composition of rotation of 45° about y-axis, followed by a dilation with the factor $k = \sqrt{2}$. (06)

(C) CLO3, BL3

Find the basis for kernel and range of the linear transformation $T: R^2 \to R^2$ defined by $T(x,y) = (2x - y, -8x + 4y)$ and verify the dimension theorem. (06)

Q.5 **Solve the following** **(18)**

(A) CL04, BL3

Determine algebraic and geometric multiplicity of each eigenvalue and corresponding eigen vector for the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. (06)

(B) CLO4, BL3

Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. (06)

(C) CLO4, BL3	Find a matrix P that diagonalizes $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ and hence find A^{10}.	(06)
Q.6	**Solve the following**	**(14)**
(A) CLO3, BL3	Consider the basis $B = \{u_1, u_2\}$ and $B' = \{v_1, v_2\}$ for R^2, where $u_1 = (1,-1)$, $u_2 = (0,6)$, $v_1 = (2,1)$ and $v_2 = (-1,4)$. (i) Find the transition matrix from B' to B. (ii) Find the transition matrix from B to B'.	(07)
(B) CLO4, BL3	Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is similar to diagonal matrix and hence find its diagonal and modal matrix.	(07)

Nirma University

Institute of Technology

Supplementary Examination (SPE), February - 2025

B. Tech. in All Programmes, Semester-I

1MH101CC22 Mathematics - I

Roll/Exam No.:________________ Supervisor's initial with date ________________

Time: 3 Hours | Max. Marks: 100

Instructions:

1. Attempt all questions.
2. Figures to the right indicate full marks.
3. Use Section-wise separate answer book.

Section-I

Q.1 **Answer the following:** **(18)**

(A) CLO1, BL3 — Find the rank of matrix $A = \begin{bmatrix} 1 & -2 & -2 & 1 & 1 \\ -2 & 4 & 4 & -2 & -1 \\ 4 & -8 & -8 & 4 & 3 \\ 3 & -6 & -5 & 2 & 1 \end{bmatrix}$ by reducing into row echelon form. (06)

(B) CLO1, BL3 — Find the value of a and b, for which the system of linear equations (06)

$$x + y + z = 6, x + 2y + 3z = 10, x + 2y + az = b$$

has unique solution.

(C) CLO1, BL3 — Solve: $3x + y = 0,\ x + 2y - z = 0,\ 5x + 5y + 2z = 0.$ (06)

Q.2 **Answer the following:** **(18)**

(A) CL02, BL3 — Give the answers with proper justification: (06)

(i) Is $V = \{(a,b)\epsilon R^2 : k(a,b) = (ka,b), k\epsilon \boldsymbol{R}\}$ together with standard vector addition and scalar multiplication, a vector space?

(ii) Is the set $W = \{a_0 + t^2 : a_0 \epsilon \boldsymbol{R}\}$ a subspace of vector space $\boldsymbol{P_2(t)}$, polynomials in t of degree at most 2 with coefficients in R?

(B) CLO2, BL3 — Determine whether $S = \{2 - 3x + x^2, 4 + x + x^2, -7x + x^2\}$ is a basis for the vector space $P_2(x)$, where $P_2(x)$ is the set of polynomials in x of degree at most 2 with coefficients in $\mathbf{R}$. (06)

(C) CLO2, BL3 — Extend the set $S = \{(2,1,1,2),(-1,1,1,-1)\}$ to the basis of vector space $\mathbf{R}^4$. (06)

Q.3 Answer the following: (14)

(A) CLO1, BL3 — Find the inverse for matrix $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ 3 & -12 & 3 & -27 \\ -2 & 10 & 2 & 24 \\ -1 & 6 & 1 & 14 \end{bmatrix}$ if possible. (07)

(B) CLO2, BL3 — For the bases $S = \{2, 2x, 2x^2\}$ and $T = \{1+x^2, 1-x^2, x+x^2\}$ of $P_2(x)$ (07)

(i) Find the transition matrix from S to T,

(ii) Find the transition matrix from T to S,

(iii) Compute $[w]_S$ for $w = 3 - 4x + 7x^2$ and

(iv) Compute $[w]_T$ using $[w]_S$.

Section-II

Q.4 Answer the following: (18)

(A) CLO3, BL3 — Let $T: M_{22} \to R$, be a linear transformation and (06)

$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$, is the basis of M_{22} such that

$T\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\} = 3, T\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\} = -4, T\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right\} = 1, T\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\} = -1.$

Determine the linear transformation T and compute the value of $T\left\{ \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \right\}$.

(B) CLO3, BL3 — Find the standard matrix of the composition of a rotation of 60^0 about x-axis, followed by a rotation of 30^0 about z-axis, followed by a contraction with the factor $k = \frac{1}{2}$ in R^3. (06)

(C) CLO3, BL3 — Let T: $\mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation defined by (06)

$T(x,y,z) = (x + 2y - z,\ y - z, x + y - 2z)$

(i) Find a basis and dimension of the range/image of T and the kernel of T

(ii) Find inverse of T (if exist).

Q.5 Answer the following: (18)

(A) CL04, BL3 — Let $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$, find the algebraic multiplicity & geometric multiplicity of each eigen values and corresponding eigen vectors. (06)

(B) CLO4, BL3 — Verify Caley-Hamilton theorem for $A=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then using it find the matrix of $A^5 - 3A^4 + A^3 - 7A^2 + 5A + I$. (06)

(C) CLO4, BL3 — Using orthogonal diagonalization, diagonalize the matrix (06)

$$A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

Q.6 Answer the following: (14)

(A) CLO3, BL3 — Consider the linear transformation $T: \boldsymbol{R}^2 \to \boldsymbol{R}^2$, defined by $T(x,y) = (x-y, -y)$ and $E = \{e_1 = (1,0), e_2 = (0,1)\}$ & $S = \{u_1 = (2,1), u_2 = (-3,4)\}$ are bases of $\mathbf{R^2}$ (07)

(i) Find the matrix A representing T relative to the basis E.

(ii) Find the matrix B representing T relative to the basis S.

(iii) Verify $B = P^{-1}AP$, where P is the change-of-basis (transition) matrix from the basis E to S.

(B) CLO4, BL3 — Find a matrix P that Diagonalize $A=\begin{pmatrix} 2 & 2 & 2 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$, and hence find A^8. (07)